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Useful Results of HCC Transform

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Abstract: In this paper, have study useful results of half canonical cosine transform of generalized function.

Keyword: Canonical transform, half canonical cosine transform, Integral transform, generalized function testing function space.

1. Introduction: Now a days, fractional integral transforms play an key role in signal processing, image reconstruction, pattern recognition, accostic signal processing [1],[2].A new generalized integral transform was obtained by Zayed[8]. Bhosale and Chaudhary [3], Chavhan S B [5],[6][7]. Had extended fractional Fourier transform to the distribution of compact support. Chavhan S B [4] had define the half canonical cosine transform as

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b\neq 0$$

Notation and terminology as per Zemanian [9],[10].This paper is organized as section 2 definition of testing function space .Section 3 definition of half canonical cosine transform. Section 4 useful results of half canonical cosine transform and lastly conclusion is stated.

2. Definition of testing function space E**:**

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$, if for each compact set. $I\subset s_a$ where $s_a=\{t:\in R^n,\ \ |t|\le a, a>0\}$ and for $k\in R^n$,

$$\gamma_{E,k}\phi(t)=\sup_{t\in I}\left|D^k\phi(t)\right|<\infty \qquad k=0,1,2,3.....$$

Note that space E is complete and a Frechet space,let E' denotes the dual space of E

3. Definition of half canonical cosine transform:

- Half canonical cosine transform of $f(t)$ is given by

$$\{HCCTf(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b\neq 0$$

$$=\sqrt{d}\,.e^{\frac{i}{2}cds^2}f(d.s) \qquad \text{for } b=0$$

Where, $K_{HC}(t,s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$

Hence half canonical cosine transform of $f(t)$ is defined as

$$\{HCCT\,f(t)\}(s)=\langle f(t),K_{HC}f(t,s)\rangle$$

Since the range of integration for the half canonical cosine transform is just $[0,\infty]$ and not $(-\infty,\infty)$ using half canonical cosine transform is more convenient than using the canonical transform to deal with the even function.

4. Useful Property of half canonical transform:

4.1 Time Reverse Property:

If $\{HCCT\,f(t)\}$ is half canonical cosine transforms of $f(t)$, $f(t)\in E'(R')$ then

$$\{HCCT\,f(-t)\}(s)=\{HCCT\,f(t)\}(s)$$

Proof : Using definition of half canonical cosine transforms of $f(t)$

$$\{HCCT\,f(t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}f(t)\,dt$$

$$\{HCCT\,f(-t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \cos\left(\frac{-s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2}f(-t)\,dt$$

Put $-t=x \qquad \therefore t=-x$

Hence $dt=-dx$, also $t\to 0\ to\infty,\ x\to\infty,0$

$$\{HCCT\,f(-t)\}(s)=-\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{\infty}^{0}\cos\left(\frac{s}{b}x\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(-x)^2}f(x)dx$$

$$\{HCCT\,f(-t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{0}^{\infty}\cos\left(\frac{s}{b}x\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(x)^2}f(x)dx$$

Replacing x by t again

$$\{HCCT\,f(-t)\}(s)=\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_{0}^{\infty}\cos\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(t)^2}f(t)dt$$

$$\therefore\{HCCT\,f(-t)\}(s)=\{HCCT\,f(t)\}(s)$$

4.2 Parity :

If $\{HCCT\,f(t)\}(s)$ is half canonical cosine transform of $f(t)\in E^1(R^1)$ then

$$\{HCCT\,f(-t)\}(s)=\{HCCT\,f(t)\}(-s)$$

4.3 Linearity Property :

If C_1, C_2 are constant and $f_1,\ f_2$ are functions of t then

$$\{HCCT[C_1f_1(t)+C_2f_2(t)]\}(s)=C_1\{HCCT\,f_1(t)\}(s)+C_2\{HCCT f_2(t)\}(s)$$

4.4) Addition property: If $\{HCCT\,f(t)\}(s)$ and $\{HCCT\,g(t)\}(s)$ are half canonical cosine transform of $f(t)$ and $g(t)$ then

$$\{HCCT[f(t)+g(t)]\}(s)=\{HCCT\,f(t)\}(s)+\{HCCT\,g(t)\}(s)$$

5) Conclusion:

In this paper half canonical cosine transforms is generalized in the form the distributional sense, we have obtained useful results for this transform are proved.

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